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Entitled

A System And Method For Graphically Analyzing Product Interactions

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A SYSTEM AND METHOD FOR GRAPHICALLY ANALYZING PRODUCT INTERACTIONS

RELATED INFORMATION:

[0001] This application claims priority under 35 U.S.C. § 119(e) to provisional application number 60/226,798, filed August 21, 2000, entitled "Method and System for Graphically Representing Customer Affinities", which is hereby fully incorporated by reference.

BACKGROUND OF THE INVENTION

Field of the Invention

[0002] The present invention relates to quantifying and using product and web-page interactions to maximize profits, and more particularly to a method for visualizing, understanding and using the interactions between products or web-pages to drive sales.

Description of the Related Art

[0003] Interactions have been largely ignored in past analysis of retail operations. Conventional wisdom is that the pricing, advertising, and display of products are the primary driver for their sales. However, according to empirical research, approximately 70% of items in both general merchandise and grocery stores undergo no price change in any given year. Therefore, sales of most items in the store may be driven by pricing decisions in other products.

[0004] Fig. 1 depicts a demand forecasting experiment conducted at two retailers. A stepwise regression was employed to select any variable available from the point-of-sale (POS) record, including the quantity sold up to 5 days in the past, the item's price, the average price at the store, and the prices of other items at

the store. The number of times that each variable was selected by a stepwise regression procedure is shown on the vertical axis. The study revealed that the prices of other items at the store tended to be the best variable for predicting any particular item's demand.

[0005] In another experiment, products with high lift-affinities, which will be defined in detail later, were physically moved together in a store. This resulted in a 30% increase in profits from those items.

[0006] The above results suggest that interactions in the retail store significantly impact profitability. Yet, until recently, few attempts have been made to comprehensively understand price and shelf-elasticity interactions between products. Quantifying and interpreting the price interactions of products is difficult because of the number of products. With as many as 100,000 products in a typical large retailer's inventory – there are up to 10 billion potential cause-effect relations. For this reason, analysis of price interactions has been limited to extremely small numbers of products. Previous work on price elasticity was limited to a few isolated products, such as less than a dozen products in the yogurt area.

[0007] Table 1 shows sales interactions between buns and biscuits and other products in the store. The table shows the probability of purchasing an item b, given a purchase of buns and biscuits.

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACHMENT	LIFT AFFINITY	% OF STORE BASKETS WITH AFFINITY ITEM	NO. OF BASKETS WITH BOTH ITEMS
WIENERS – FRANKS	18.6%	20.1%	6.19	3.1%	16408
BRATWURST	3.2%	16.3%	5.00	0.6%	2873
CONDIMENTS – CATSUP	6.5%	10.9%	3.31	2.0%	5827
REFRIGERATED CHEESE SLICES	11.4%	10.0%	3.04	3.7%	10379

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACHMENT	LIFT AFFINITY	% OF STORE BASKETS WITH AFFINITY ITEM	NO. OF BASKETS WITH BOTH ITEMS
FROZEN POTATOES	9.3%	10.0%	3.03	3.0%	8564
FRESH BONELESS BEEF	31.2%	9.5%	2.95	10.7%	26954
BEANS – BAKED	9.5%	9.3%	2.84	3.3%	8488
CANNED-PREPARED FOOD/PASTA/STEW	6.8%	8.9%	2.71	2.5%	6251
WHITE BREAD	20.5%	8.9%	2.71	7.6%	17820
BOX PREPARED FOOD MAC/CHS/HHLP	11.0%	8.8%	2.67	4.1%	10011
SNACKS – CHIPS / WAREHOUSE	34.2%	8.7%	2.67	12.8%	29638

Table 1: Top Lift Affinities for buns and biscuits

[0008] Reports like those depicted in Table 1 are lengthy (the table would continue on for hundreds of pages as it lists every interaction), one-dimensional (i.e. flat), and important relationships can be difficult to see. Relationships between three or more products, or relationships between groups of items are almost impossible to ascertain and appreciate in this format.

[0009] What is need is an improved way to graphically display the information and succinctly and precisely lay out the relationships between groups of related items and aide a user in navigating the items in a store and identify important relationships.

SUMMARY OF THE INVENTION

[0010] The present invention has been made in view of the above circumstances and has as an aspect an improved visualization method for understanding and utilizing interactions between products. A graphical representation of the affinities or "Affinity Graph" is a model of the relationships between items and depicts items as nodes and interactions as arcs.

[0011] A further aspect of the present invention is improved retail functions, such as assortment, pricing, promotion, and shelf layout by employing different affinity statistics.

[0012] An additional aspect of the present invention is a system for graphically displaying and optimizing interaction data between items in a retail setting, the system comprising a general purpose computer having memory capable of operating pursuant to instructions from an algorithm, wherein the algorithm further comprises the steps of loading the interaction metric between items into memory, optimizing placement of nodes and edges pursuant to the interaction metric, and generating a graphical representation of the nodes and edges with corresponding interaction metrics.

[0013] A method for graphically illustrating interactions between items offered for sale, the method including importing interaction metrics between items into a memory device for the optimum placement of nodes and edges pursuant to the interaction metrics. Generating a graphical representation of the nodes and edges with the corresponding interaction metrics. Additional aspects and advantages of the invention will be set forth in part in the description which follows, and in part will be obvious from the description, or may be learned by practice of the invention. The aspects and advantages of the invention will be realized and attained by means of the elements and combinations particularly pointed out in the appended claims.

[0014] To achieve these and other advantages and in accordance with the purpose of the present invention, as embodied and broadly described, the present

invention can be characterized according to one aspect the invention as comprising a method for graphically displaying and optimizing interaction data between items in a retail setting, the method comprising evaluating the interaction correlations between items, optimizing placement of nodes and edges pursuant to the interaction correlations, and generating a graphical representation of the nodes and edges with corresponding interaction correlations.

[0015] It is to be understood that both the foregoing general description and the following detailed description are exemplary and explanatory only are not restrictive of the invention, as claimed.

BRIEF DESCRIPTION OF THE DRAWINGS

[0016] The accompanying drawings, which are incorporated in and constitute a part of this specification, illustrate one several embodiments of the invention and together with the description, serve to explain the principles on of the invention.

[0017] Fig. 1 depicts the results of a cross-price elasticity study;

[0018] Fig. 2 depicts a link between an interaction matrix and its graphical representation of the present invention;

[0019] Fig. 3 depicts an example of an affinity graph wherein a node is labeled with unit profit and an edge is labeled with a probability of purchasing b given the occurrence of a of the present invention;

[0020] Fig. 4 is a graphical representation of an affinity report for Buns and Biscuits of Table 3 of the present invention;

[0021] Fig. 5 illustrates a correlation between pasta and homestyle spaghetti sauce at a basket level of the present invention;

[0022] Fig. 6 depicts an affinity graph for a school/stationary cluster of the present invention;

[0023] Fig. 7 illustrates lift affinities for school products of the present invention;

[0024] Fig. 8 depicts a correlation setup between several products evaluated in the present invention;

[0025] Figs. 9A and 9B illustrate items for sale that have an underlying tendency to be purchased in concert;

[0026] Fig. 10 is an illustration of an affinity graph of the present invention; and

[0027] Figs. 11 – 12 depict various affinity graphs of the present invention.

DETAILED DESCRIPTION OF THE INVENTION

[0028] Reference will now be made in detail to the present embodiments of the invention, examples of which are illustrated in the accompanying drawings. Wherever possible, the same reference numbers will be used throughout the drawings to refer to the same or like elements.

[0029] In accordance with the invention, the present invention can be characterized according to one aspect the invention as comprising a method for graphically displaying and optimizing interaction data between items in a retail setting, the method comprising quantifying the interaction(s) between items, optimizing placement of nodes (items) and edges (interactions), and generating a graphical representation of the nodes and edges.

THE AFFINITY GRAPH

[0030] Fig. 2 illustrates the link between the interaction matrix and its graphical representation. The *i*th row of the matrix records the effect of *i* on every other item *j* in the store.

[0031] Fig. 3 depicts and exemplary affinity graph where the node is labeled with unit profit, and the edge is labeled with a probability of purchasing b given a. The graph illustrates that if Gravy is purchased, there is a 20% chance of purchasing Stuffing. Examples of additional affinity graphs are provided in Figs. 2, 3, 4, and 5, respectively.

[0032] Table 2 illustrates how flat one-dimensional data in a report can be transformed into a graph. The list of attachments is the probability of a product being purchased given a purchase of bananas. Graphically, for the wieners-franks node there will emerge a set of out-arrows, each pointing to different attachment items. Informally, this shows the probability of wieners-franks "causing" the purchase of these other items.

[0033] The reverse attachments are represented as a set of arrows pointing towards the wieners-franks. Informally, these arrows may indicate the probability of each product "driving" the sale of wieners-franks.

[0034] Thus these conventional tabular reports actually showed only a "slice" of the total relationships between products, centered around the interactions of one item. A full graph, on the other hand, has the potential of revealing a hundreds of significant relationships and interaction-clusters.

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WHITE BREAD	20.5%	8.9%	2.71	7.6%	17820
BOX PREPARED	11.0%	8.8%	2.67	4.1%	10011

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACHMENT	LIFT AFFINITY	% OF STORE BASKETS WITH AFFINITY ITEM	NO. OF BASKETS WITH BOTH ITEMS
FOOD-MAC/CHS/HHLP					
SNACKS – CHIPS/WAREHOUSE	34.2%	8.7%	2.67	12.8%	29638

Table 2: Affinities for buns and biscuits

GRAPH CONSTRUCTION

[0035] Figure 4 is a pared down graph depicting affinities for only several items, most graphs are much larger, by virtue of there being upwards of 100,000 products in a large retail store. This means that there are potentially 100,000 nodes and the square of 100,000 results in there being 10 billion arcs. Displaying this many arcs would be unwieldy.

[0036] Fortunately there is an entire field which studies the problem of laying out graphs, called *Graph Drawing*. Graph Drawing algorithms have attracted much attention as of late because of their application in circuit design. A new circuit design needs to be laid out on a 2D wafer, so that the total number of wire crossings are kept to a minimum. Also, the surface area of the circuit should be kept low as possible to again minimize on material used and also operational speeds and tolerances. The same techniques for optimizing layout according to some predefined criteria, can be applied in making non-circuit graphs easier to read and navigate.

[0037] Graph drawing algorithms manipulate placement of nodes and edges until the defined criteria is met, if possible, that a list of criteria is met. The criteria often includes the following parameters to be met:

[0038] minimizing the number of crossings between edges;

[0039] minimizing spacing between linked nodes;

[0040] minimizing area of graph;

[0041] maximize horizontal and vertical symmetries; and

[0042] the angle between two edges onto a node should be greater than a predefined constant.

[0043] The present invention normally takes one of two general approaches to draw the retailing graphs. The first is the use of force-directed methods which attempt to spread the graph using spring-like forces into symmetrical patterns. The second is hierarchical methods, which attempt to order the graph into a directional flow.

Graphs generated using force equations

[0044] Force-directed methods have a long history in applied mathematics (e.g. see Tutte, W. (1963), How to Draw a graph, *Proceedings of the London Mathematical Society*, Vol. 13, No. 3, pp. 743-768.). These methods use the idea of attraction and repulsion between nodes, and between edges, in order to optimize graph layout. The basic model represents edges as springs and nodes as charged particles which repel each other. The force in the x-dimension on any vertex is equal to an equation similar to:

[0045] $$F_X(v) = \sum_{(u,v)\in E} k_{uv}^{(1)}(d(p_u,p_v) - l_{uv})\frac{x_v - x_u}{d(p_u,p_v)} + \sum_{(u,v)\in V} \frac{k_{uv}^{(2)}}{d(p_u,p_v)^2}\frac{x_v - x_u}{d(p_u,p_v)}$$

[0046] where l_{uv} is the natural or zero energy length of the spring between u and v, $k_{uv}^{(1)}$ is the stiffness of the spring (the larger this value, the closer the spring should be to its ideal distance), $p_v=(x_v,y_v)$ is the position of node v, $k_{uv}^{(2)}$ gives the strength of repulsion between nodes u and v. (Battista, G., Eades, P., Tamassia, R. and Tolis, I. (1999) *Graph Drawing: Algorithms for the*

visualization of graphs, Prentice Hall, NJ.). Many variants on this energy equation are known in the art.

[0047] Finding the minimum energy configuration of the graph falls to numerical algorithms. A variety of optimization and heuristic methods have been developed. One excellent heuristic is to move each node in the direction opposite to its force. This amounts to gradient descent on the global force function, since

[0048]
$$F_x = \sum_{v \in V} F_x(v)$$
$$\frac{\partial F_x}{dF_x(v)} = F_x(v)$$

[0049] A gradient descent procedure on *x* would become

[0050]
$$x_v(t+1) = \alpha\left[x_v(t) - \frac{\partial F_x}{\partial F_x(v)}\right]$$
$$= \alpha\left[x_v(t) - F_x(v)\right]$$

[0051] The algorithm can terminate after a maximum number of node adjustments, or after the change in force improvement has dropped below a threshold, or the change in node position has dropped below a certain threshold. Reflexive edges can be handled by inserting two dummy nodes to create a well-spaced self-loop.

Hierarchical graphs

[0052] Hierarchical graphs try to convert the graph into an ordered, top-down tree with nodes at the same distance from the root on the same level. A simple hierarchical algorithm following Battista, G., Eades, P., Tamassia, R. and Tolis, I. (1999) *Graph Drawing: Algorithms for the visualization of graphs*, Prentice Hall, N.J. is provided below:

[0053] Firstly let each y_v equal *–depth(v)*, which is the number of nodes between the root node and *v*. In order to planarize the tree, ensure that the left to right order of two nodes *u* and *v* is the same as the parents of those nodes, *parent(u)* and *parent(v)*. Next, traverse the graph depth-first and assign each node encountered its order in the traversal (eg. The 5^{th} node encountered is assigned 5). This will be the starting *x*-position of each node. A compaction will then be performed on the graph. Recursively move down each subtree of the graph. For each pair of subtrees, move them together until they are c units apart.

[0054] Non-top-down links can be accomodated by reversing the direction of the link, running the algorithm, and then reversing the direction of the arrow after the final result is produced. If edges span more than one level, dummy nodes are introduced to the edge. Reflexive edges are dealt with by inserting a self-loop using two dummy nodes. The above algorithm has a remarkable running time, being able to run in O(*n*) time where *n* is the number of nodes in the graph.

Thresholding to provide visibility

[0055] Even with graph layout software, affinity graphs are often still too densely inter-connected to be easily readable. Therefore, a further step is needed. It has been determined that it is possible to threshold the graph to only display nodes or edges if they meet certain criteria. This improves the visibility of the graph considerably.

[0056] One common threshold method is to show edges where lift is greater than a particular threshold. This restricts the graph to only the most "significant" edges.

[0057] An additional threshold shows nodes or edges where they have been purchased more than a certain number of times.

[0058] For instance, there might only be 3 times that Ft. Soda was bought, and on one of those occasions, Canned Soup happened to be in the basket also. Ft. Soda has so few sales that the fact Canned Soup was bought one occasion is as likely due to random chance as due to any true correlation between the products. However, without thresholding an edge will be created showing that there is a 33% chance of Canned Soup being bought given a sale on Ft. Soda. In practice a lot of "low support, high weight" links make their way into the graph, and these can mis-represent relationships between products. Thresholding eliminates these spurious edges. Euler's theorem also decreases the number of crossings, and thereby increases readability. Ideally the user should have control over the level of thresholding via a pop-up window attached to the graph visualization GUI.

Display parameters

[0059] Parameters affecting the graph include:

[0060] Node statistic;

[0061] Edge statistic; (some function of the two nodes)

[0062] Time-period range (the start and end of time-period);

[0063] Customer-base which the affinities are generated for (i.e. all or top-ranked customers);

[0064] Level of item-aggregation; and

[0065] Level of transaction-aggregation

[0066] For example, to exclusively show the affinities which will be operating in an upcoming "back to school" week, affinities can be generated for just that week

(i.e. time-period of analysis). Appendix A provides a complete list of edge, node, item and transaction-aggregation levels, and the affinities which result.

EXAMPLES OF GRAPHS FOR RETAILING APPLICATIONS

[0067] The following are some examples of interaction graphs with particular application to retailing. Note that each graph is defined by the selection of edge measure, node measure, time-period, variable-pair, and aggregation level.

[0068] Correlation graphs show complimentary and substitute products;

[0069] Attachment graphs show cross-sale probabilities;

[0070] Cross-elastic price graph show price cannibalization;

Complimentary and Supplementary products

[0071] Generally, in economics it is important to identify is which products are substitutes and compliments. For example, Wieners and ketchup are complimentary, while Tide and Arm and Hammer laundry detergent are supplimentary.

[0072] The correlation between basket sales of two products can help reveal whether they behave as complimentary or supplementary products. An interaction metric IM(i,j) of this type is:

[0073]
$$IM(i,j) = \frac{\sum_{t=1}^{N}\left((qty_{ti} - E[qty_i]) \cdot (qty_{tj} - E[qty_j])\right)}{\sqrt{\sum_{t}^{N}\left((qty_{ti} - E[qty_i])^2 \cdot (qty_{tj} - E[qty_j])^2\right)}}$$

[0074] A positive correlation indicates that a customer typically buys the two products together, and a negative indicates that the customer either buys one product or the other.

[0075] Fig. 5 depicts the correlation between pasta and homestyle spaghetti sauce. At the basket level, Fig. 5 illustrates that sales are correlated for both items, and therefore they are complimentary products.

Cross-sell Probabilities

[0076] Pr(*j*|*i*) gives the probability of purchasing *j* given *i*. For example, if a customer bought Ft. Soda Refill 15 times, and on 5 of those occasions, the customer also bought a newspaper, then there is a 1 in 3 chance of a newspaper being bought whenever someone buys a Ft. Soda Refill. We can define this interaction metric as .

[0077]
$$\begin{aligned} IM(i,j) &= Attachment(i,j) = \Pr(qty_j > 0 \mid qty_i > 0) \\ &= \frac{\sum_{t=1}^{N}\left(sign(qty_{ti}) \wedge sign(qty_{tj})\right)}{\sum_{t=1}^{N} sign(qty_{ti})} \end{aligned}$$

To identify attachments which are "greater or lower than a normal purchase baseline", a different metric is defined. *Lift* Affinity is one such metric and is defined as the affinity divided by the probability of the item appearing in any basket.

[0078]
$$\begin{aligned} IM(i,j) &= \frac{\Pr(qty_j > 0 \mid qty_i > 0)}{\Pr(qty_j > 0)} \\ &= \frac{\Pr(qty_j > 0 \wedge qty_i > 0)}{\Pr(qty_j > 0) \cdot \Pr(qty_j > 0)} \end{aligned}$$

[0079] The number which results is the number of times greater than random, which *j* is purchased after purchasing *i*. A lift-affinity less than 1 means the probability of purchasing *j* is *1/lift._affinity* times "lower" than normal. For example, a lift of 1/4 means the probability of purchasing *j* is 4 times lower than normal.

[0080] Fig. 6 depicts affinities sorted in order of lift-affinity. Fig. 6 further illustrates that for the "products which make sense" all are sorted to the top. For yogart, the top lift affinities are pop-tarts, apples, bananas. The bottom lift-affinities are smoking accessories, pepporoni pizza and fish fry. This implies that consumers who buy yogart tend to buy healthy foods, rather than smoking accessories, fish fry or pepporoni pizza.

[0081] Fig. 7 illustrates a basic affinity graph for a retailer, wherein the cluster is in school/stationary

Top 10 Attachments for Yogurt

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACH-MENT	AFFINITY LIFT	PERCENT OF STORE BASKETS WITH AFFINITY ITEM	NUMBER OF BASKETS WITH BOTH ITEMS
MILK & MILK BY-PRODUCTS	53.2%	7.2%	2.14	24.9%	48656
BANANAS	42.0%	10.9%	3.28	12.9%	38615
BEVERAGE - SOFT. DRINKS	32.8%	5.2%	1.51	21.7%	30189
CEREAL - READY-TO-EAT	30.8%	10.9%	3.24	9.5%	29081
SNACKS - CHIPS/WAREHOUSE	28.1%	7.4%	2.19	12.8%	26092
SALAD VEGETABLES-CABBAGE/LETT.	27.8%	10.6%	3.16	8.8%	26332
VARIETY VEGETABLES	23.9%	9.3%	2.76	8.7%	22834

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACH-MENT	AFFINITY LIFT	PERCENT OF STORE BASKETS WITH AFFINITY ITEM	NUMBER OF BASKETS WITH BOTH ITEMS
FRESH BONELESS BEEF	23.0%	7.3%	2.16	10.7%	21604
EGGS	21.9%	8.5%	2.52	8.7%	20652
LUNCHEON MEAT	20.9%	9.1%	2.66	7.8%	19687

Table 3

[0082] A simple list of the top affinities for category yogurt results in the most frequently purchased items, milk, bananas, soft drinks, cereal, chips, being listed. Table 3 illustrates lift-affinities for the same category, which depict categories which have a "higher than normal" probability of being purchased once yogurt is purchased. These affinities tell a much more vivid story, such as pop-tarts, cream, and other items have an unusually high chance of being purchased if yogurt is purchased Tables 4 and 5 illustrate the top lift and bottom lift affinities for Yogurt.

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACH-MENT	AFFINITY LIFT	PERCENT OF STORE BASKETS WITH AFFINITY ITEM	NUMBER OF BASKETS WITH BOTH ITEMS
BREAKFAST SPECIALTIES POPTART	10.4%	12.4%	3.44	2.9%	10786
SOFT. FRUITS	11.1%	11.4%	3.41	3.3%	10699
APPLES	16.4%	11.8%	3.39	4.7%	16304
BANANAS	42.0%	10.9%	3.28	12.9%	38615
CEREAL – READY-TO-EAT	30.8%	10.9%	3.24	9.5%	29081
FROZEN CONC. JUICES	10.9%	11.3%	3.19	3.3%	11215

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACH-MENT	AFFINITY LIFT	PERCENT OF STORE BASKETS WITH AFFINITY ITEM	NUMBER OF BASKETS WITH BOTH ITEMS
SALAD VEGETABLES-CABBAGE/LETT.	27.8%	10.6%	3.16	8.8%	26332
VARIETY FRUIT	11.9%	10.8%	3.12	3.7%	11908
GRAPES	12.6%	10.5%	3.08	4.1%	12131
BOX PREPARED FOOD-MAC/CHS/HHLP	12.6%	10.5%	3.02	4.1%	12553
FROZEN VEGETABLES	10.1%	10.5%	2.96	3.3%	10465

Table 4: Top lift-affinities for Yogurt

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACH-MENT	AFFINITY LIFT	PERCENT OF STORE BASKETS WITH AFFINITY ITEM	NUMBER OF BASKETS WITH BOTH ITEMS
TAMPONS	0.0%	0.3%	0.07	0.4%	5
PUFF PASTRIES	0.0%	0.3%	0.07	0.4%	5
ANTACID TABLETS & LIQUID	0.0%	0.2%	0.07	0.5%	5
TOYS	0.0%	0.3%	0.06	0.6%	10
COLD & SINUS TABLETS-COLD MED.	0.0%	0.3%	0.06	0.7%	11
FRIDAY FISH FRY DINNER	0.0%	0.3%	0.06	0.5%	5
BROWNIES / BARS	0.0%	0.3%	0.06	0.5%	5
PEPPERONI	0.0%	0.3%	0.06	0.5%	5
BEER - POPULAR PRICED	0.1%	0.2%	0.05	1.8%	46

AFFINITY ITEM	ATTACHMENT	REVERSE ATTACH-MENT	AFFINITY LIFT	PERCENT OF STORE BASKETS WITH AFFINITY ITEM	NUMBER OF BASKETS WITH BOTH ITEMS
NON-CIGARETTES TOBACCO	0.0%	0.1%	0.03	1.4%	5

Table 5: Bottom lift-affinities for Yogurt

Price cannibalization coefficients

[0083] Price cannibalization reveals whether one product cannibalizes another based on price. For instance, if the sales of "I can't believe its not butter" increase when the price of butter goes up, then "I can't believe its not butter" cannibalizes butter. The direction and degree of price cannibalization can be determined by looking at the correlation between price of product a and sales of product b. If the sales of b go up in response to an increase in price of a, then b cannibalizes a. If there is a negative correlation, then product b is also detrimentally impacted by the price increase in a, so it is a complimentary item and is defined as follows:

[0084]
$$IM(i,j) = \frac{\sum_{t=1}^{N}\left((price_{ti} - E[price_i]) \cdot (qty_{tj} - E[qty_j])\right)}{\sqrt{\sum_{t}^{N}\left((price_{ti} - E[price_i])^2 \cdot (qty_{tj} - E[qty_j])^2\right)}}$$

[0085] Other metrics such as price elasticity ε_{ij} may also be used. Assumingprice elasticity is estimated using a linear demand function, $qty_j = w_{ij} * price_i + constant$, ε_{ij} can be defined as:

[0086]

$$
\begin{aligned}
IM(i,j) &= \frac{\partial qty_i}{\partial price_j} \cdot \frac{price_{Ti}}{qty_{Tj}} \\
&= \frac{\left(price_i^{T} \cdot qty_j\right)}{\left(price_i^{T} \cdot price_i\right)} \cdot \frac{price_{Ti}}{qty_{Tj}} \\
&= \frac{\sum_{t=1}^{T}\left(price_{ti} \cdot qty_{tj}\right)}{\sum_{t=1}^{T}\left(price_{ti} \cdot price_{tj}\right)} \cdot \frac{price_{Ti}}{qty_{Tj}}
\end{aligned}
$$

[0087] Cannibalization coefficients provide a rich view of product interactions. Most people are familiar with products competing with each other. Cannibalization analysis reveals that this is only one of four possible relationships, the others are mutual cooperation, competition, exploited cooperation and parasitism.

[0088] Cooperation occurs when a sale on one product benefits another, and vica versa. Parasitism (and exploitation) occurs when a sale on A benefits B, but a sale on B then proceeds to cannibalize A. For example, gravy and stuffing are mutual cooperators. Potato chips could be parasitic on regular sales in the store.

[0089] Understanding these relationships can provide valuable information, especially to manufacturers. For instance, it would typically not be desirable to endorse a company whose product is cannibalizing your product. Likewise, it is possible to locate unexpected allies who you can coordinate advertising sharing and cross-sale programs.

[0090] A second advantage of cannibalization analysis is that it is capable of being computable at the timeseries level. Since prices, advertisements, and shelf layouts do not normally change faster than a day, basket-level data can be forgone in favour of aggregated daily-level data. Longer intervals of 30-days can work even better. Because most of the items in the store are “slow movers” (i.e. have more than 50% of their days recording no-sale) the ordinary sales timeseries can be recoded into a 30-day moving aggregate. After this

transformation, each point of this timeseries is read as the total units to be sold in the next 30 days.

[0091] Figure 7 shows a 411 day, 30-day moving sum for "Peter Pan crunchy peanut butter 18oz". Around day 65 both "Smuckers strawberry jam" and "Bush's baked beans vegetarian" went on sale, represented by a drop in their prices. Simultaneously there was a demand increase in peanut butter. Therefore, the correlation statistic reveals that price of Smucker's strawberry jam and Bush's baked beans are both negatively correlated with demand in peanut butter. Therefore, the drops in prices of these items increase the sales of Crunchy Peanut Butter.

[0092] One problem with price cannibaliation is that several items could have had price reductions at the same time, and all would show correlations, and be considered possible drivers. Therefore, this analysis is most reliable when a number of price changes have occurred in a non-correlated fashion, when a large amount of data is available, and when extraneous factors have been controlled or taken into account.

[0093] Table 7 shows a tabular price cannibalization report for the category "Applesauce". This reveals that "Melons" are parasitic on "Applesauce". This means that if Applesauce goes on sale, it results in an increase in purchases of other items such as Melons. However, if Melons go on sale, they detract from the sales of Applesauce. Perhaps when Melons go on sale, this results in cherry pickers who purchase large quantities of Melons without purchasing basic grocery items.

Coefficient of attraction

[0094] The four-relationships can also be summarized as "cooperation" or "competition" by averaging the cross-effect of a on b, and b on a. In matrix notation this computation is defined as follows:

[0095] $$IM(i,j)^{*} = IM(i,j) + IM(j,i)$$

Matrix pos	ITEM	Designation	My cross-deriv	Their cross-deriv
213	MELONS	parasite	-0.68	0.00
148	FROZEN NOVELTIES	parasite	-0.66	0.04
30	BEVERAGE-WATERS/NEW AGE/SPORT	ally	-0.60	-0.12
300	SOFT FRUITS	ally	-0.51	-0.01
301	SOUP	competitor	0.51	0.15
187	INSECT SPRAYS-RAID/CUTTERSPRAY	parasite	-0.47	0.08
85	COCOA MIXES	competitor	0.46	0.15
101	COUGH SYRUP	competitor	0.46	0.12
90	COLD & SINUS TABLETS-COLD MED.	competitor	0.43	0.12
309	SPRAYS-RUBS & LOZENGES/NASAL	competitor	0.43	0.07
222	MISC. GENERAL MERCHANDISE	ally	-0.43	-0.07
255	PICNIC - SPEC. SALADS	parasite	-0.41	0.09
102	CRACKERS - SALTINE & GRAHAMS	competitor	0.40	0.09
307	SPONGE CAKE	parasite	-0.40	0.14
106	DANISH / COFFEE CAKE	ally	-0.40	0.00

Table 7: Parasite-host report for Applesauce

[0096] **(using *Corr(pricei,qtyj)* matrix)**

APPLICATIONS

Application 1: Increase sales by manipulating drivers

[0097] The first application helps increase the sales of a product. Normally if retailers need to move a product, they might think about putting it on sale, or creating a display of the product. But now we can provide an alternative. We can put other items which might be thought of as "drivers" on sale, and then allow them to drive increased consumption in the target product. This may be accomplished by finding items which have high probabilities onto the product. Intuitively these are the strongest arrows pointing into product *j*.

[0098] i : **max** $\Pr(j|i)$

Application 2: Increase profit by Cross-selling

[0099] If a product *i* is already going on sale (perhaps the buyer got a special offer on it), what other products should be advertised with it? To maximize sales, the affinity graph is used to find the highest probability affinity $\Pr(j|i)$ and is denoted as

[0100] j . **max** $\text{Attachment}(i,j)$ = **max** $\Pr(qty_j>0 \mid qty_i>0)$

[0101] To maximize profit, it is desirable to find the highest probability multiplied by unit profit $P(j)$,

[0102] j . **max** $\Pr(qty_j>0 \mid qty_i>0)*P(j)$

[0103] Graphically, this is analogous to looking at the arrows coming out from item *i*, and finding the strongest affinity.

[0104] Fig. 10 depicts an example of how various driver items relate to a product.

[0105] Fig. 11 depicts that the finding of cross-sales items corresponds to finding items being pointed to by *j* with a high weight.

Application 3: Promoting products which are likely to pull in second-order sales

[0106] Given *n* products which are being advertised. What *m* other products should be advertised to maximize predicted profit/sales? This can be done by calculating the base contribution of the item plus the contributions from the affinities of the product. The total second-order affinity-related contribution of each product can be calculated by simply summing profit multiplied by probability of sale.

[0107] *ProfitContribution(j)* = $\Pr(j)\cdot(\pi_j + \sum_{k=1}^{N}\Pr(k \mid j)\cdot\pi_k)$

[0108] *SalesContribution(j)* = $\Pr(j)\cdot(1 + \sum_{k=1}^{N}\Pr(k \mid j))$

[0109] Fig. 12 illustrates how this can maximize profit. A Game console, and another product have already been chosen to be advertised. Another product needs to be selected which will maximize profit. If NFL Jackets are chosen, these would pull in Jeans and the product already being advertised, resulting in a total profit of contribution of NFL Jackets (20) + probability of secondary sale multiplied by profit of secondary sale, or a total contribution of 22.4.

[0110] However, if Jeans were selected (contribution 10), several other items would be pulled in, including an expensive Game Console (contribution 100). The expected contribution for that item is 37.0. Therefore, although NFL Jackets have a better unit profitability, Jeans would be a better product to promote

because they have better connections to other cliques in the store as shown in Fig. 18.

[0111] *j* **max** *ProfitContribution(j)* where $j \notin m$

[0112] There are other criteria that can be maximized also. One criteria might be to increase traffic in as many categories of the store as possible. For this, a *DiversityContribution* metric defined below can be used:

[0113] $$DiversityContribution(i) = \frac{1}{|C|}\sum_{c \in C} sign(\sum_{j \in c} \Pr(j \mid i) > mean(\Pr(c)) + \alpha \cdot stdev(\Pr(c)))$$

[0114] *C* is the set of categories. The *mean(*Pr*(c)+α.stdev(*Pr*(c)))* term means that the category is counted as a "1" if there exists an item in the category (*j*) which *i* triggers with probability greater than the mean plus standard deviations of the normal probability of inclusion in that category. Overall an item has a high diversity score if it has a high probability of triggering several different categories. Other ways to score diversity are possible.

Application 4: Promote items omitted from cliques

[0115] A clique defines a set of items which are commonly purchased together. Therefore, if a customer comes into the store and purchases 3 products {*a*,*b*,*c*} out of the 4 items in a known clique {*a*,*b*,*c*,*d*}, then either:

[0116] the customer does not know about *d*;

[0117] the customer has purchased *d* on a previous visit (this data is known, thereby ruling out this possibility);

[0118] the customer does not want to buy *d* for some reason, making this customer different to the population norm;

[0119] the customer is buying *d* elsewhere.

[0120] Of all these possibilities, the most enticing is that the customer is puchasing *d* elsewhere, or does not know about *d*. In these particular cases, this dropped item can be marketed in an attempt to encourage the customer to purchase this product, and the present invention should be able to increase his/her basket size.

[0121] By looking for item omissions in common cliques, the potential to either increase basket size for that customer is made possible, by encouraging the customer to buy the omitted products, and possibly capture additional business from a competitor.

Application 5: Promote dominating items to capture a clique

[0122] A Dominating item is an item which, if purchased, has the greatest expected profit based on affinities in the clique, or greatest expected sales in the clique, based on affinities. Thus, the dominating item "dominates" or "captures" the clique better than any other item. The dominating item could be the "key" to winning the clique.

[0123] Dominating items are easy to identify. Be simply applying the *SalesContribution* and *ProfitContribution* calculations to each item in a clique, where this contribution is only calculated for the clique itself. Once this has been done, there can be listed the most influential items in order.

[0124] $$DiversityContribution(i) = \frac{1}{|C|}\sum_{c \in C} sign\left(\sum_{j \in c} \Pr(j \mid i) > mean(\Pr(c)) + \alpha \cdot stdev(\Pr(c))\right)$$

[0125] CliqueSalesContribution(i,c) = SalesContribution(i) where only $k \in c$ are counted

[0126] CliqueProfitContribution(i,c) = ProfitContribution(i) where only $k \in c$ are counted

Application 6: Special offers on "Bridging items" to cross-sell customers into new categories

[0127] Bridging items are items which have strong connections to more than one clique. Bridging items can be useful for planning advertisements, linking one theme with another, and can be used to capture multiple cliques.

[0128] Bridging items between cliques are calculated using the *DiversityContribution* metric, where *C* is changed to be the set of cliques, rather than the set of categories. Therefore, an item receives a high score for being a bridging item if it spans many cliques (rather than categories).

[0129] It will be apparent to those skilled in the art that various modifications and variations can be made in the present invention and in construction of this invention without departing from the scope or spirit of the invention. Other embodiments of the invention will be apparent to those skilled in the art from consideration of the specification and practice of the invention disclosed herein. It is intended that the specification and examples be considered as exemplary only, with a true scope and spirit of the invention being indicated by the following claims.